

HOMOTOPICAL MODELS OF TYPE THEORY

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Homotopy Type Theory (HoTT) is based on a homotopical interpretation of Martin-Löf type theory, which in turn justifies some powerful new logical principles such as higher inductive types and Voevodsky's celebrated univalence axiom [Uni13]. Recently several major advances have been made, resolving some of the open questions that arose in the decade since its original formulation. Specifically, the question of the constructive character of the new logical principles has been largely resolved in work by Coquand and others, using methods involving cubical sets, a variation on the conventional simplicial approach to homotopy theory [CCHM18]. The relation between the cubical and simplicial models has recently been clarified in work by the author and others [ACCRS19]. Modern treatments of homotopy theory often rely on the notion of an ∞ -topos, a higher categorical generalization of Grothendieck's influential concept of a topos. Shulman has recently shown that HoTT has models in all ∞ -toposes [Shu19], verifying a long-standing conjecture. This talk will review the basic ideas and survey some of these recent developments.

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