APPLICATIONS OF PROOF THEORY TO CORE MATHEMATICS: RECENT DEVELOPMENTS

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In this talk we survey some recent developments in the project of applying proof-theoretic transformations to obtain new quantitative and qualitative information from given proofs in areas of core mathematics such as nonlinear analysis, convex optimization and geodesic geometry ([4, 5]). We will discuss some of the following items:

(1) The recent extraction of uniform rates of convergence for the ε-capture in the Lion-Man game in the context metric spaces X satisfying a suitable 'betweennes property' ([9]). Here a low complexity rate of convergence is extracted from a proof that made iterated use of sequential compactness arguments (i.e. arithmetical comprehension). The extraction also replaced the strong assumption of the compactness of X by its boundedness.

(2) In [1], we extracted the first explicit moduli of uniform continuity from noneffective continuity proofs for concepts of proximal maps in uniformly convex Banach spaces. It turned out, that in order to get a modulus which (as desired) is independent of the scalar involved one has to modify the previously suggested definition of such maps giving rise to a new definition.

(3) Recently, [3] showed that certain uses of weak sequential compactness can be replaced by classically false uniform boundedness arguments (also called bounded collection) which can be eliminated from convergence proofs with no complexity contribution. This approach is used e.g. in [2] to obtain rates of metastability for a strongly convergent version of the proximal point algorithm in Hilbert space.

(4) In [8], it is shown how the assumption of metric regularity gives low complexity rates of convergence for algorithms which in general do not have computable rates of convergence. In the particular case of the uniqueness of the solution such an approach is used in [10] for a wide range of algorithms computing zeros

of accretive set-valued operators as used for abstract Cauchy problems.

(5) An arithmetization of a highly noneffective convergence proof for the computation of so-called sunny nonexpansive retractions has recently led to effective bounds in [11] and is used in [7] to obtain metastability of a strongy convergent form of the proximal point algorithm in Banach spaces.

(6) A polynomial rate of convergence in Bauschke's celebrated solution of the 'zero displacement conjecture' has recently been extracted from Bauschke's proof which heavily uses the machinery of maximal monotone operators ([6]).

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